



Gauge Dependence of the Critical Dynamics at the Superconducting Transition

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The critical dynamics of superconductors in the charged regime is reconsidered within field-theory. For the dynamics the Ginzburg-Landau model with complex order parameter coupled to the gauge field suggested earlier [Lannert et al. Phys. Rev. Lett. **92**, 097004 (2004)] is used. Assuming relaxational dynamics for both quantities the renormalization group functions within one loop approximation are recalculated for different choices of the gauge. A gauge independent result for the divergence of the measurable electric conductivity is obtained only at the weak scaling fixed point unstable in one loop order where the time scales of the order parameter and the gauge field are different.

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1. Introduction

The nature of the static phase transition in superconductors was an open question for decades, since due to the large correlation length of existing superconducting materials the effect of critical fluctuations was hard to observe in the vicinity of the critical temperature T_c . Moreover it was unclear if the phase transition is of first or second order and, if it is of second order, to which universality class the transition belongs. The appearance of high- T_c superconducting compounds with short correlation lengths [1] made the critical region of superconductors experimentally accessible. In turn this leads to comparison of the experiment with analytical results in order to establish the critical properties of superconductors and their universality class.

From the theoretical point of view the static critical properties are now well understood. The theoretical model for the description of the static critical properties was formulated in [2] and contains, besides the two component ($n = 2$) order parameter (OP) and its fourth order interaction term, a minimal coupling to a gauge field (GF) due to the charged character of the OP (macroscopic wave function of the Cooper pair). The coupling of the OP to the GF introduces an essential difference to the superfluid phase transition, where the OP is uncharged.

While for type-I superconductors fluctuation effects are weak and a mean-field analysis can be applied, the situation with type-II superconductors is more complicated, since here fluctuations of the OP cannot be neglected. The first renormalization group (RG) analysis in one-loop approximation [2] lead to the conclusion that a stable fixed point (FP) and therefore a second order phase transition only exists for OP dimensions n larger than 365.9. For OP dimension $n = 2$ no stable FP was found and the runaway solution was interpreted as a weak first order phase transition. A two loop order calculation of the renormalization group equations within field theory indicated the possibility of a continuous phase transition for $n = 2$ [3] if certain resummations are performed. This has been investigated further and led to the following picture [4]: There are four FPs, two uncharged ones known from the standard Ginzburg-Landau-Wilson (GLW) model and two charged ones. Depending on the initial (background) conditions for the flow of the fourth order coupling and the coupling to the charge (defining the value of the Ginzburg parameter κ) one obtains a

runaway flow or the flow reaches the stable charged FP. The other charged FP is reached starting on the separatrix (defined by $\kappa = \kappa_c$) separating the attraction region of the charged FP from the runaway region. Physically this FP reached on the separatrix describes tricritical behavior indicating that a tricritical point separates the first order transition of the superconductors of type I from the second order transition for superconductors of type II. Results of duality arguments [7] and Monte Carlo calculations [8] are in agreement with this picture, which is also supported by experiment [9, 10].

An important question in calculating critical properties like the critical exponents is their dependence on the gauge used for the GF. Physically observable quantities should be independent of the specific gauge used. This has been shown in one loop order [5]. However non observable quantities might depend on the gauge. Thus the critical exponents ν or α of the penetration length λ (proportional to the correlation length ξ of the OP correlation function at the charged FP) and the specific heat respectively should be independent of gauge, whereas the critical exponent of the static OP correlation function turns out to be gauge dependent [6].

The issue of dynamical critical properties was less studied. Experimental investigations give no consistent picture of the dynamical critical exponent z (for references see Ref. [11]) obtained from measurements of the electrical conductivity. The values of the dynamical critical exponents found vary between 1.5 and 2.3.

Theoretical predictions of z are mainly based on using the results of known universality classes (model A, C, E or model F, for a review see [12]). Model A is the simplest model assuming a relaxation equation for the OP without coupling to other conserved densities, whereas model C contains a coupling to a diffusion equation for a conserved density (only relevant if the specific heat is diverging). Models E and F define the universality class of the critical dynamics of the superfluid transition in ^4He . Dynamic equations from the vortex-loop model [13, 14] indicate a relation to that universality class. So far no systematic derivation of dynamical equations with mode coupling terms derived from Poisson bracket relations using for instance methods described in [15] has been performed.

Monte Carlo simulations [13] in the limiting cases of large and small values of the Ginzburg parameter found values of the dynamical exponent of 2.7 and 1.5 respectively. Using the vortex model [11] for the superconducting transition the dynamic critical exponent has been calculated analytically in the two limits mentioned above. The resulting exponents were $5/2$ and $3/2$ respectively and they were related to the dynamic exponents obtained in simulations. A discussion [16] arised on the reason of the deviation of the Monte Carlo results from the expected value of model A, which would lead to a dynamical exponent of roughly 2. Recently in Ref. [14] for *extreme* type II superconductors, where the *uncharged* FP describes the static behavior, it was argued using scaling and duality arguments that $z = 3/2$ exactly.

A concrete dynamical model defined by equations of motion for the OP and the GF has been presented in Ref. [17]. Both equations are of relaxational character and the essential parameter is the ratio $w = \Gamma_\psi/\Gamma_A$ of the kinetic coefficients Γ_ψ of the OP and Γ_A of the GF. A dynamic FP, w^* finite, implying *strong scaling* with a common dynamical critical exponent z for the OP and the GF in the Feynman gauge (one adds a quadratic term in the divergence of the GF to the static functional) has been found in one loop order. However the question of the gauge dependence of the dynamical exponents was not addressed in Ref. [17].

Another important issue not considered so far in the discussion of the dynamical critical behavior concerns the time scales entering the problem. One has to discriminate between the time scale for the OP and its dynamic critical exponent z_ψ and the time scale of the GF and its dynamic critical exponent z_A . They are defined by the characteristic frequencies of the dynamic correlation functions for the OP and the GF respectively

$$\omega_\psi \sim k^{z_\psi} g_\psi(k\xi) \quad \omega_A \sim k^{z_A} g_A(k\xi). \quad (1)$$

g_ψ and g_A are scaling functions which are finite and nonzero at the superfluid transition T_c .

The dynamic critical exponents are different [18] if *weak scaling* holds otherwise one says the dynamic correlation functions obey *strong scaling*. So far in the discussion the last case was

assumed. Even if for the uncharged FP the dynamics would be in the universality class of model E, it is the *weak scaling* FP which is stable [19].

Only one of these dynamical exponents is observable namely z_A entering the frequency dependent diverging electric conductivity

$$\sigma(\xi, k = 0, \omega) \sim \xi^{(z_A - 2 + \eta_A)}, \quad (2)$$

$\eta_A = 4 - d$ is the anomalous dimension of the GF independent of the gauge [20]. This implies that the dynamical critical exponent z_A has to be gauge independent. No such condition holds for z_ψ . If therefore it turns out that z_ψ is gauge dependent *strong scaling* cannot hold.

In this paper we apply the field-theoretical RG approach in the minimal subtraction scheme and dimensional regularization to the dynamical model of Ref. [17] in order to investigate the gauge dependence of the critical dynamics. It turns out that the dynamical exponent z_ψ of the OP is gauge dependent and a gauge independent value for z_A is only obtained in the *weak scaling* FP $w^* \rightarrow \infty$, which however has been found unstable. Therefore one has to question the physical relevance of the one loop result. This goes together with the problems in statics of the Ginzburg-Landau model at the one loop order level.

2. Model

Static critical properties of charged superconductors in d dimensional space are described by the Abelian Higgs model with the static functional [2]:

$$\mathcal{H} = \int d^d x \left\{ \frac{1}{2} \dot{r} |\vec{\psi}_0|^2 + \frac{1}{2} \sum_{i=1}^{n/2} |(\nabla - i \dot{e} \mathbf{A}_0) \psi_{0,i}|^2 + \frac{\dot{u}}{4!} (|\vec{\psi}_0|^2)^2 + \frac{1}{2} (\nabla \times \mathbf{A}_0)^2 \right\}. \quad (3)$$

with the complex n -component OP $\vec{\psi}_0$ (generalized to $n/2$ -components, the superconductor being the case $n = 2$) and the d -dimensional GF \mathbf{A}_0 . The bare parameter \dot{r} is proportional to the distance from T_c and the bare coupling \dot{e} is the effective charge. \dot{u} is the usual fourth order OP coupling. In all subsequent calculations we add to the static functional the contribution

$$\mathcal{H} = -\frac{1}{2\zeta} (\nabla \cdot \mathbf{A}_0), \quad (4)$$

which allows to choose the gauge. In the limit $\zeta \rightarrow 0$ the Landau gauge is recovered, while for system with $\zeta \rightarrow 1$ the Feynman gauge is achieved.

The OP and the GF are nonconserved quantities thus their dynamical behavior may be described by relaxation equations. Such a set of equations of motion has been suggested [17]:

$$\begin{aligned} \frac{\partial \psi_{0,i}}{\partial t} &= -2\dot{\Gamma}_\psi \frac{\delta \mathcal{H}}{\delta \psi_{0,i}^+} + \theta_i; \\ \frac{\partial \psi_{0,i}^+}{\partial t} &= -2\dot{\Gamma}_\psi \frac{\delta \mathcal{H}}{\delta \psi_{0,i}} + \theta_i^+; \\ \frac{\partial A_{0,\alpha}}{\partial t} &= -\dot{\Gamma}_A \frac{\delta \mathcal{H}}{\delta A_{0,\alpha}} + \theta_\alpha. \end{aligned} \quad (5)$$

The OP relaxes with the kinetic coefficient Γ_ψ while the GF relaxes with the kinetic coefficient Γ_A . The stochastic forces in (5) are related to the kinetic coefficients and satisfy the relations:

$$\begin{aligned} <\theta_i(\mathbf{x}, t)\theta_j^+(\mathbf{x}', t')> &= 4\dot{\Gamma}_\psi \delta(\mathbf{x} - \mathbf{x}') \delta(t - t') \delta_{ij}, \\ <\theta_i(\mathbf{x}, t)> &= 0, \\ <\theta_\alpha(\mathbf{x}, t)\theta_\beta(\mathbf{x}', t')> &= 2\dot{\Gamma}_A \delta(\mathbf{x} - \mathbf{x}') \delta(t - t') \delta_{\alpha\beta}, \\ <\theta_\alpha(\mathbf{x}, t)> &= 0. \end{aligned} \quad (6)$$

The indices i, j adopt the values $1, \dots, n/2$, while the indices α, β adopt the values $1, \dots, d$.

As it was pointed out in Ref. [17] the equation for the GF in (5) can be derived from Maxwell's equations in their low-frequency form. In this case the inverse transverse coefficient $\mathring{\Gamma}_A^{-1}$ for the GF can be identified with the bare normal electrical conductivity.

We study this critical dynamics by applying the Bausch-Janssen-Wagner approach [21] of dynamical field-theoretical RG. In this approach, the critical behavior is considered on the basis of long-distance and long-time properties of the Lagrangian incorporating the features defined by the dynamical equations of the model. In order to keep the powers in the interaction terms low an auxiliary OP density $\tilde{\psi}$ and an auxiliary GF \tilde{A} have been introduced. The dynamic equations defined by (3)-(6) are then described by the unrenormalized Lagrangian $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$ separated into a Gaussian part

$$\begin{aligned} \mathcal{L}_0 = & \int d^d x dt \left[-4\mathring{\Gamma}_\psi \sum_{i=1}^{n/2} \tilde{\psi}_{0,i}^+ \tilde{\psi}_{0,i} - 2\mathring{\Gamma}_A \tilde{A}_0^2 + \sum_{i=1}^{n/2} \tilde{\psi}_{0,i}^+ \left(\frac{\partial}{\partial t} + \mathring{\Gamma}_\psi (\mathring{r} - \Delta) \right) \psi_{0,i} + \right. \\ & \left. \sum_{i=1}^{n/2} \tilde{\psi}_{0,i} \left(\frac{\partial}{\partial t} + \mathring{\Gamma}_\psi (\mathring{r} - \Delta) \right) \psi_{0,i}^+ + \tilde{A}_0 \cdot \left(\frac{\partial \mathbf{A}_0}{\partial t} + \mathring{\Gamma}_A \left(\nabla \left(1 - \frac{1}{\zeta} \right) (\nabla \cdot \mathbf{A}_0) - \Delta \mathbf{A}_0 \right) \right) \right], \end{aligned} \quad (7)$$

and an interaction part

$$\begin{aligned} \mathcal{L}_1 = & \int d^d x dt \left[\mathring{\Gamma}_\psi \frac{\dot{u}}{3!} |\psi|^2 \sum_{i=1}^{n/2} (\tilde{\psi}_{0,i}^+ \psi_{0,i} + \tilde{\psi}_{0,i} \psi_{0,i}^+) + 2\mathring{\Gamma}_\psi i \dot{e} \mathbf{A}_0 \cdot \sum_{i=1}^{n/2} (\tilde{\psi}_{0,i}^+ \nabla \psi_{0,i} - \tilde{\psi}_{0,i} \nabla \psi_{0,i}^+) + \right. \\ & + \mathring{\Gamma}_\psi i \dot{e} (\nabla \cdot \mathbf{A}_0) \sum_{i=1}^{n/2} (\tilde{\psi}_{0,i}^+ \psi_{0,i} - \tilde{\psi}_{0,i} \psi_{0,i}^+) + \mathring{\Gamma}_A \frac{1}{2} i \dot{e} \tilde{A}_0 \cdot \sum_{i=1}^{n/2} (\psi_{0,i} \nabla \psi_{0,i}^+ - \psi_{0,i}^+ \nabla \psi_{0,i}) + \\ & \left. \mathring{\Gamma}_\psi \dot{e}^2 \mathbf{A}_0^2 \sum_{i=1}^{n/2} (\tilde{\psi}_{0,i}^+ \psi_{0,i} + \tilde{\psi}_{0,i} \psi_{0,i}^+) + \mathring{\Gamma}_A \dot{e}^2 \tilde{A}_0 \cdot \mathbf{A}_0 |\psi|^2 \right]. \end{aligned} \quad (8)$$

3. Perturbative expansion and renormalization

3.1. Vertex functions

In order to proceed the vertex functions for the OP and the GF are calculated in one loop expansion. We keep the general structure of the two point vertex functions [12] and separate the static contributions. The calculation leads to the following general form of the OP vertex function

$$\mathring{\Gamma}_{\psi \tilde{\psi}^+} = -i\omega \mathring{\Omega}_{\psi \tilde{\psi}^+} + 2\mathring{\Gamma}_\psi \mathring{\Gamma}_{\psi \psi^+}^{st}. \quad (9)$$

In one loop order the dynamical OP function $\mathring{\Omega}_{\psi \psi^+}$ reads

$$\begin{aligned} \mathring{\Omega}_{\psi \tilde{\psi}^+} = & 1 + 4\dot{e}^2 \mathring{\Gamma}_\psi \int \frac{1}{(r+(k+k')^2)k'^2(-i\omega+\mathring{\Gamma}_\psi(\mathring{r}+(k+k')^2)+\mathring{\Gamma}_A k'^2)} \left(k^2 - \frac{(kk')^2}{k'^2} \right) \\ & + \dot{e}^2 \mathring{\Gamma}_\psi \int \frac{1}{(\mathring{r}+(k+k')^2)k'^2(-i\omega \mathring{\zeta} + \mathring{\zeta} \mathring{\Gamma}_\psi(r+(k+k')^2) + \mathring{\Gamma}_A k'^2)} \left(\frac{(2k+k')k'}{k'^2} \right), \end{aligned} \quad (10)$$

whereas the static OP vertex function is given by

$$\begin{aligned} \mathring{\Gamma}_{\psi \psi^+}^{st} = & \frac{1}{2} \left\{ \mathring{r} + k^2 + \frac{n+2}{6} \dot{u} \int \frac{1}{(\mathring{r}+k'^2)} + \dot{e}^2 (d-1+\mathring{\zeta}) \int \frac{1}{k'^2} - \right. \\ & \left. 4\dot{e}^2 \int \frac{1}{(r+(k+k')^2)k'^2} \left(k^2 - \frac{(kk')^2}{k'^2} + \mathring{\zeta} \frac{(2k+k')k'}{k'^2} \right) \right\}. \end{aligned} \quad (11)$$

The same general structure holds for the GF vertex function

$$\mathring{\Gamma}_{A\tilde{A}}^{\alpha\beta} = -i\omega\mathring{\Omega}_{A\tilde{A}}^{\alpha\beta} + \mathring{\Gamma}_A\mathring{\Gamma}_{AA}^{st\alpha\beta}, \quad (12)$$

with the one loop expression

$$\mathring{\Omega}_{A\tilde{A}}^{\alpha\beta} = \delta^{\alpha\beta} + \mathring{\Gamma}_A \frac{n}{2} \int \frac{(k+2k')^\alpha(k+2k')^\beta}{(\mathring{r}+k'^2)(\mathring{r}+(k+k')^2)[-i\omega+\mathring{\Gamma}_\psi(\mathring{r}+k'^2)+\mathring{\Gamma}_\psi(r+(k+k')^2)]}, \quad (13)$$

for the dynamic GF function and

$$\begin{aligned} \mathring{\Gamma}_{AA}^{st\alpha\beta} &= k^2 \left(\delta^{\alpha\beta} - \frac{k^\alpha k^\beta}{k^2} + \frac{1}{\zeta} \frac{k^\alpha k^\beta}{k^2} \right) \\ &+ n\dot{e}^2 \int \frac{1}{\mathring{r}+k'^2} \left(\delta_{\alpha\beta} - \frac{1}{2} \frac{(k+2k')^\alpha(k+2k')^\beta}{\mathring{r}+(k+k')^2} \right), \end{aligned} \quad (14)$$

for the GF static vertex function.

3.2. Renormalization and field theoretic functions

In order to get finite results we perform the renormalization within the minimal subtraction scheme introducing renormalization factors leading to the renormalized parameters. The renormalization factors of the GLW part of the static functional are introduced as usual

$$\psi_{0,i} = Z_\psi^{1/2}\psi_i, \quad \mathring{r} - \mathring{r}_c = Z_\psi^{-1}Z_{rr}, \quad \dot{u} = \kappa^\varepsilon \mathcal{A}_d^{-1}Z_uZ_\psi^{-2}u, \quad (15)$$

where the shift of the phase transition temperature \mathring{r}_c has been taken into account. κ represents the free wave vector scale and $\varepsilon = 4 - d$. We also introduced the usual geometric factor

$$\mathcal{A}_d = \Gamma\left(1 - \frac{\epsilon}{2}\right)\Gamma\left(1 + \frac{\epsilon}{2}\right)\frac{\Omega_d}{(2\pi)^d},$$

where Ω_d is the surface of the d -dimensional unit sphere. The additional renormalization factors due to the presence of the GF and its coupling to the OP are introduced as

$$A_{0,\alpha} = Z_A^{1/2}A_\alpha, \quad \dot{e}^2 = \kappa^\varepsilon \mathcal{A}_d^{-1}Z_{e^2}Z_\psi^{-1}Z_A^{-1}e^2, \quad \dot{\varsigma} = Z_\varsigma^{-1}Z_{A\varsigma}. \quad (16)$$

From Ward identities one derives the relations [5]

$$Z_{e^2} = Z_\psi, \quad Z_\varsigma = 1. \quad (17)$$

which show that only one additional (for the GF) renormalization constant with respect to the GLW model appears.

In dynamics two additional renormalization factors are needed which are

$$\tilde{\psi}_{0,i} = Z_\psi^{1/2}\tilde{\psi}_i, \quad \tilde{A}_{0,\alpha} = Z_{\tilde{A}}^{1/2}\tilde{A}_\alpha. \quad (18)$$

For the kinetic coefficients no new factors are necessary

$$\mathring{\Gamma}_\psi = Z_{\Gamma_\psi}\Gamma_\psi = Z_\psi^{1/2}Z_{\tilde{\psi}}^{-1/2}\Gamma_\psi, \quad (19)$$

$$\mathring{\Gamma}_A = Z_{\Gamma_A}\Gamma_A = Z_A^{1/2}Z_{\tilde{A}}^{-1/2}\Gamma_A, \quad (20)$$

where the second relations are due to the structure of the vertex functions (see (9) and (12)).

The renormalization factors calculated in one loop order for the statics are

$$Z_\psi = 1 + (3 - \varsigma)\frac{e^2}{\varepsilon}, \quad Z_A = 1 - n\frac{e^2}{6\varepsilon}, \quad (21)$$

$$Z_r = 1 + \frac{n+2}{6} \frac{u}{\varepsilon} - \varsigma \frac{e^2}{\varepsilon}, \quad Z_u = 1 + \frac{n+8}{6} \frac{u}{\varepsilon} + \frac{18}{u} \frac{e^4}{\varepsilon} - 2\varsigma \frac{e^2}{\varepsilon}, \quad (22)$$

as presented in [3]. The dynamic renormalization factors are

$$Z_{\tilde{\psi}} = 1 - (3 - \varsigma) \frac{e^2}{\varepsilon} - 2 \frac{e^2 \varsigma}{\varepsilon} \frac{w}{1+w}, \quad (23)$$

$$Z_{\tilde{A}} = 1 + n \frac{e^2}{2\varepsilon} \left(\frac{1}{3} - \frac{1}{w} \right), \quad (24)$$

where we have introduced the time scale ratio

$$w = \frac{\Gamma_\psi}{\Gamma_A}, \quad (25)$$

whose FP value w^* determines if *strong* ($w^* \neq 0$ or ∞) or *weak scaling* ($w^* = 0$ or ∞) holds.

From the Z -factors one obtains the ζ -functions leading to the β -functions which determine the FPs. The critical exponents describing the critical properties are then expressed by the values of the ζ -functions at the stable FPs. We use the unified definition

$$\zeta_a(\{\alpha\}, \varsigma) = - \frac{d \ln Z_a}{d \ln \kappa}. \quad (26)$$

for all ζ -functions, where a denotes either any model parameter $\{\alpha\} = \{u, e^2, \Gamma_\psi, \Gamma_A, w\}$ or any density $\psi, \tilde{\psi}, A, \tilde{A}$. The static ζ -functions following from (21) and (22) read

$$\zeta_\psi = (3 - \varsigma)e^2, \quad \zeta_A = -n \frac{e^2}{6}, \quad (27)$$

$$\zeta_r = \frac{n+2}{6}u - \varsigma e^2, \quad \zeta_{e^2} = (3 - \varsigma)e^2, \quad (28)$$

$$\zeta_u = \frac{n+8}{6}u - 2\varsigma e^2 + 18 \frac{e^4}{u}. \quad (29)$$

The ζ -function of the gauge parameter follows from (16) and (17) as $\zeta_\varsigma = 0$. The dynamic ζ -functions follow from (23). They are

$$\zeta_{\tilde{\psi}} = -(3 - \varsigma)e^2 - 2e^2 \varsigma \frac{w}{1+w}, \quad \zeta_{\tilde{A}} = n \frac{e^2}{2} \left(\frac{1}{3} - \frac{1}{w} \right), \quad (30)$$

and the ζ -functions for the kinetic coefficients are obtained from the above relations (19) and (20)

$$\zeta_{\Gamma_\psi} = \frac{1}{2}(\zeta_\psi - \zeta_{\tilde{\psi}}), \quad \zeta_{\Gamma_A} = \frac{1}{2}(\zeta_A - \zeta_{\tilde{A}}). \quad (31)$$

Inserting the one loop results (30) they read

$$\zeta_{\Gamma_\psi} = e^2 \left(3 - \frac{\varsigma}{1+w} \right), \quad \zeta_{\Gamma_A} = -n \frac{e^2}{2} \left(\frac{1}{3} - \frac{1}{2w} \right). \quad (32)$$

According to Eqs. (25) and (31), the ζ -function of the time scale ratio w is then found as

$$\zeta_w = \zeta_{\Gamma_\psi} - \zeta_{\Gamma_A} = e^2 \left(3 - \frac{\varsigma}{1+w} + \frac{n}{2} \left(\frac{1}{3} - \frac{1}{2w} \right) \right). \quad (33)$$

4. Fixed points and exponents

The behavior of the model parameters under renormalization is described by the flow equations

$$\ell \frac{d\alpha_i}{d\ell} = \beta_{\alpha_i}(\{\alpha\}, \varsigma), \quad (34)$$

where the right hand sides of the equations are determined by appropriate β -functions (the index i runs over the set of parameters). The β -functions are generally defined as

$$\beta_{\alpha_i}(\{\alpha_i\}, \varsigma) = \alpha_i [-c_i - p_i \zeta_\psi - q_i \zeta_A + \zeta_{\alpha_i}], \quad (35)$$

The coefficients c_i , p_i and q_i follow from the general renormalization of α_i as used above

$$\dot{\alpha}_i = \kappa^{c_i} \mathcal{A}_d^{-(p_i+q_i)/2} Z_\psi^{-p_i} Z_A^{-q_i} Z_{\alpha_i} \alpha_i. \quad (36)$$

The FPs $\{\alpha^*\}$ are defined by the zeros of the right hand sides of Eqs. (34). A FP is stable if all stability exponents ω_i are positive. The stability exponents ω_i are defined by the eigenvalues of the matrix $\partial \beta_{\alpha_i} / \partial \alpha_j|_{\{\alpha\}=\{\alpha^*\}}$.

4.1. Statics

The one-loop static β -functions appear to be independent from the gauge determined by the parameter ς and are the same as in Ref. [2].

$$\beta_u = \left(-\varepsilon u - 6e^2 u + \frac{n+8}{6} u^2 + 18e^4 \right), \quad \beta_{e^2} = e^2 \left(-\varepsilon + \frac{n}{6} e^2 \right). \quad (37)$$

These β -functions have four FPs: (i) the Gaussian FP with $u^* = e^{*2} = 0$, (ii) the FP of the uncharged XY-model ($u^* = 6\varepsilon/(n+8)$, $e^* = 0$), (iii) the tricritical FP with $u^* = 0$, $e^{*2} = 6\varepsilon/n$ and (iv) the charged FP with both couplings $u^* \neq 0$ and $e^{*2} = 6\varepsilon/n$ nonzero. In one loop order the charged FP exists only for $n > 365.9$ [2]. However in higher loop order and using summation procedures a charged FP is also found for $n = 2$ [22]. In one loop order for $n < 365.9$ starting with $e^2 \neq 0$ and $u \neq 0$ the flow escapes to $e^2 \rightarrow e^{*2}$ and $u \rightarrow \infty$.

Critical exponents are calculated at the stable accessible FP. Expressions for the correlation length critical exponent ν , the specific heat exponent α and the pair correlation function critical exponent η are as follows:

$$\nu^{-1} = 2 - \zeta_r(u^*, e^*, \varsigma) + \zeta_\psi(u^*, e^*, \varsigma) = 2 - \frac{n+2}{6} u^* + 3e^{*2}; \quad (38)$$

$$\alpha = 2 - d\nu; \quad (39)$$

$$\eta = \zeta_\psi(u^*, e^*, \varsigma) = (3 - \varsigma)e^{*2}. \quad (40)$$

All other static critical exponents may be found from scaling relations. Since the FP values u^* and e^* are gauge independent this is also valid for ν and α although the gauge ς appears in the ζ -functions Eq. (38) explicitly. In contrast this dependence on the gauge remains in η (see Eq. (40)). All the other exponents become gauge dependent since they are in any case related to η by the scaling laws. However note that only at the charged FP ($e^{*2} \neq 0$) a gauge dependence enters the ζ -functions. This holds in all orders of the loop expansion due to the structure of the interaction part of the Lagrangian (8).

From the renormalization of ς in Equ. (16) the flow of the gauge parameter reads

$$\ell \frac{d\varsigma}{d\ell} = \varsigma \zeta_A = \varsigma^2 \frac{n}{6} e^2, \quad (41)$$

reaching zero at the charged FP. This means only the transverse gauge is invariant under renormalization.

4.2. Dynamics

The FP values for the time scale ratio can be found from the zeros of the corresponding β -function

$$\beta_w = w(\zeta_{\Gamma_\psi} - \zeta_{\Gamma_A}) = e^2 w \left(3 - \varsigma + \varsigma \frac{w}{1+w} + \frac{n}{2} \left(\frac{1}{3} - \frac{1}{2w} \right) \right). \quad (42)$$

These zeros depend in one loop order on the value of the minimal coupling (charge) e at the stable static FP. Only for non-zero charge a specific value for the possible FP is found in this order (w has to be positive)

$$w^* = \frac{n - 12(3 - \varsigma) + \sqrt{(n - 12(3 - \varsigma))^2 + 24n(18 + n)}}{4(18 + n)} \quad (43)$$

The corresponding stability exponent ω_w

$$\omega_w = \frac{\partial \beta}{\partial w} = e^{*2} w^* \left(\frac{\varsigma}{(1 + w^*)^2} + \frac{n}{4w^{*2}} \right) \quad (44)$$

turns out to be positive and thus the finite FP is stable. Another FP is the infinite FP. In order to study its stability we introduce

$$\rho = \frac{w}{1 + w} \quad (45)$$

mapping the range of w into the interval $[0, 1]$. The corresponding β -function reads

$$\beta_\rho = \rho(1 - \rho)e^2 \left(3 - \varsigma + \varsigma\rho + \frac{n}{2} \left(\frac{1}{3} - \frac{1 - \rho}{2\rho} \right) \right) = (1 - \rho)e^2 \left((3 - \varsigma)\rho + \varsigma\rho^2 + 5\frac{n}{12}\rho - \frac{n}{4} \right). \quad (46)$$

The FP $\rho^* = 1$ corresponding to the infinite FP $w^{*-1} = 0$ however is unstable with the stability exponent $\omega_\rho = -(1 + 18/n)\varepsilon$.

The dynamical critical exponents for the OP and the GF are calculated from

$$z_\psi = 2 + \zeta_{\Gamma_\psi}(u^*, e^*, w^*, \varsigma), \quad z_A = 2 + \zeta_{\Gamma_A}(u^*, e^*, w^*). \quad (47)$$

Whereas an explicit gauge dependence is found in ζ_{Γ_ψ} this is not the case for ζ_{Γ_A} at least in one loop order. It is expected that this property holds in all orders. At the charged FP ($e^{*2} = 6\varepsilon/n$) in general the exponents take the values

$$z_\psi = 2 + \frac{18}{n}\varepsilon - \varsigma \frac{6}{n} \frac{\varepsilon}{1 + w^*}; \quad (48)$$

$$z_A = 2 - \varepsilon + \frac{3\varepsilon}{2w^*}. \quad (49)$$

Since w^* depends on the gauge for the finite FP, *the dynamical exponent of the GF, z_A , depends on the gauge in contradiction to the physical requirement that observable quantities should be gauge independent*. Only in the case of the infinite FP a gauge independent value is possible. Then the OP exponent would be finite

$$z_\psi = 2 + \frac{18}{n}\varepsilon, \quad (50)$$

and also independent of the gauge but different from z_A . However the infinite FP is not stable in one loop order. It should be noted that for the gauge $\varsigma = 1$ the results of [17] are reproduced. It should also be remarked that in one loop order the calculation of the field theoretic functions is simpler in the transverse gauge ($\varsigma = 0$) due to the observation that less graphical contributions in the loop expansion are nonzero [5]. The gauge independence of the static FP values for the minimal coupling and fourth order coupling has not been proven to our knowledge. The finite FP value of the dynamical time scale w is already gauge dependent in one loop order, as has been shown here, leading to a gauge dependent critical dynamic exponent of the GF. This in our opinion would hold for every order for the **finite nonzero** FP value w^* . At the uncharged FP both dynamical exponents would be independent of the gauge.

5. Conclusion and Outlook

We have demonstrated for the dynamical model suggested by Lannert, Vishveshwara and Fisher [17] that the OP dynamical exponent is gauge dependent. Therefore also the exponent for the divergence of the electric conductivity is gauge dependent at the stable *strong scaling* FP where the dynamical critical exponents of the OP, z_ψ , and the GF, z_A , are the same. A way out in a higher loop order calculation is only possible when the stability of the *strong scaling* FP is changed and the infinite *weak scaling* FP becomes the stable one. As one knows from other examples one loop order calculations may not be conclusive and lead to results which have to be taken with care. In higher order perturbation expansion the stability of the FPs may be changed. A two loop calculation may clarify the situation. Recent progress [24] in the nonperturbative version of dynamic renormalization theory applied to this more complicated model would be also worthwhile.

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